

The rise of a tensor instability in Eddington-inspired gravity

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In this work an extension to Eddington's gravitational action is analyzed. We consider the tensor perturbations of a FLRW space-time in the Eddington regime in where the tensor mode is linearly unstable deep and the resulting modifications to Einstein regime are quite strong.

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Introduction.- One of the greatest jigsaws in the current physics research is to understand the starting point to general relativity. The Einstein-Hilbert action has been usually the initial core of gravitational theories, but an alternative idea was proposed by Eddington in Ref. [1], where the affine connection $\Gamma^\mu_{\alpha\beta}$ is the fundamental field. Stylishly, the two suggestions are equivalent by a factor of Λ . Moreover, Eddington proposal is incomplete because it doesn't include a matter sector. At this point an alternative theory of gravity was proposed in Ref. [2] where the matter fields are introduced inside the gravitational action in a Palatini form

$$S_{EBI}[g, \Gamma, \Psi] = \frac{2}{\kappa} \int d^4x \left[\sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)|} - \lambda \sqrt{g} \right] + S_m[g, \Psi], \quad (1)$$

where $\kappa = 8\pi G$, Ψ denotes any additional matter fields. $R_{\mu\nu}$ is the symmetric Ricci tensor constructed with Γ . This action can reproduce Eddington's original action at large values of κR and Einstein's at small values. Additionally, in this new theory the Eddington regime arise in the very early Universe and it may have led to a minimum scale factor. Hence, and more significantly, in this regime seems we can prevent the formation of cosmological singularities. As an enthusiastic proposal there are several literatures that proof an interesting physics, for example in Ref. [3] was showed the possibility to test the Eddington corrections to Newtonian gravity using Solar physics and in Ref. [4] the test was employed around compact rotating sources. The kindness of this new theory gives the possibility to reexpressed as a bigravity theory as was demonstrated in Ref. [5].

Instability in the Eddington Born-Infeld Theory.- Our main point is to consider a perturbed homogeneous and isotropic space-time by choosing the following two metrics

$$g_{\mu\nu} dx^\mu dx^\nu = -a^2 d\eta^2 + a^2 (\delta_{ij} + h_{ij}) dx^i dx^j, \quad (2)$$

$$q_{\mu\nu} dx^\mu dx^\nu = -X^2 d\eta^2 + Y^2 (\delta_{ij} + \gamma_{ij}) dx^i dx^j, \quad (3)$$

where a , X and Y are solely functions of conformal time, η . Here we shall work in the transverse traceless (TT) gauge, which leaves only the tensor modes in perturbations, i.e $\partial_i h^{ij} = \partial_i \gamma^{ij} = 0$. After straightforward calculations in where we found the background equations from Eq.(1) and the perturbed field equations using Eq.(2) we have that even though the tensor perturbations in both the metric and the auxiliary metrics are multiplied by different conformal factors, they are identical in this theory as we presented in Ref. [6]. Furthermore, even in the Einstein regime, where $X = Y = a$, we find that γ_{ij} is non trivial and completely locked to the behaviour of h_{ij} , which evolution equation is given by

$$h''_{ij} + \left(3 \frac{Y'}{Y} - \frac{X'}{X} \right) h'_{ij} + \left(\frac{X}{Y} \right)^2 k^2 h_{ij} = 0. \quad (4)$$

We now wish to see how this system evolves in the different regimes. In the Einstein regime we find that the evolution is indistinguishable from Einstein gravity, even though the auxiliary metric is perturbed and present. It is in the Eddington regime that we find novel behaviour for a two different values of κ :

- Case with $\kappa > 0$. In Ref. [7] it was shown a set of equations that resurrects another of Eddington's ideas of doing away with beginning and instead have the Universe indefinitely loitering in stasis in the distant past. To proceed we reexpress our background quantities in terms of conformal time, η and the evolution equation for the tensor mode is

$$h''_{ij} + 2\alpha \frac{\exp(\alpha\Delta\eta)}{[1 - \exp(\alpha\Delta\eta)]} h'_{ij} + \frac{\exp(\alpha\Delta\eta)}{[1 - \exp(\alpha\Delta\eta)]} k^2 h_{ij} = 0. \quad (5)$$

Clearly we found an instability in the Eddington regime, in the asymptotic past when $\Delta\eta \rightarrow -\infty$.

- Case with $\kappa < 0$. We found that, if one choses a closed, positively curved spatial metric, it is possible to construct an oscillating (or *Phoenix Universe*)[7] which undergoes an indefinite number of cycles. Such a model should, in principle allow us to study the evolution of perturbations through the various cycles and shed light on some of the issues that have been raised in the study of cyclic cosmologies. As we calculated in our previous work, Ref. [6], the evolution equation for the tensor mode is given by

$$h''_{ij} + \frac{2}{\eta} h'_{ij} + \frac{k^2}{3\beta^2\eta^2} h_{ij} = 0, \quad (6)$$

As in the case of previous case we find an instability this time at the bounce and the solutions blow up rendering such a space-time unstable to tensor mode perturbations.

Conclusions.- In this work we learn that traceless/transverse modes may play an unexpected role and should be included if possible in gravitational theories. Indeed by allowing more general perturbations it should also be possible trace the effect of nonlinear evolution of the tensor modes (coupled to radial modes) to search if the singularity can be stabilized in the nonlinear regime. It is remarkable to mentioned that exist a couple of analysis of perturbations in Ref. [8] where did not find such any instability in the scalar sector and this might be a hint that it is the particular form of the Eddington inspired Born-Infeld theory that gives rise to such interesting behaviour.

Last but not least, our analysis clearly hints at the possibility that interesting effects might arise in these

theories in regions of density and curvature and gives us a new way of looking at the generation and evolution of cosmological perturbations.

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